The previous chapter discussed whole numbers and decimals. This chapter looks at fractions, numbers, like decimals, that can be used to represent parts of a whole. Fractions and decimals are two ways of representing the same quantity. Fractions are used in business and our personal lives.

2.1 Basics of Fractions

Objectives

1. Recognize types of fractions.
2. Convert mixed numbers to improper fractions.
3. Convert improper fractions to mixed numbers.
4. Write a fraction in lowest terms.
5. Use the rules for divisibility.

A fraction represents part of a whole. Fractions are written in the form of one number over another, with a line between the two numbers, as in the following.

\[
\frac{5}{8}, \frac{1}{4}, \frac{9}{7}, \frac{13}{10}
\]

The number above the line is the numerator, and the number below the line is the denominator. In the fraction \(\frac{1}{3}\), the numerator is 2 and the denominator is 3. The denominator is the number of equal parts into which something is divided. The numerator tells how many of these parts are needed. For example, \(\frac{2}{3}\) is "2 parts out of 3 equal parts," as shown in the figure to the left.

Objective 1 Recognize types of fractions. If the numerator of a fraction is smaller than the denominator, the fraction is a proper fraction. Examples of proper fractions are \(\frac{2}{3}, \frac{3}{4}, \frac{15}{10}\), and \(\frac{1}{4}\). A fraction with a numerator greater than or equal to the denominator is an improper fraction. Examples of improper fractions are \(\frac{12}{12}, \frac{13}{10}, \) and \(\frac{5}{3}\). A proper fraction has a value less than 1, while an improper fraction has a value greater than or equal to 1.

To write a whole number as a fraction, place the whole number over 1; for example, \(7 = \frac{7}{1}\) and \(12 = \frac{12}{1}\). The sum of a fraction and a whole number is a mixed number. Examples of mixed numbers include \(5\frac{1}{2}\) (a short way of writing \(5 + \frac{1}{2}\)), \(3\frac{3}{4}\), and \(9\frac{5}{6}\). A mixed number can be converted to an improper fraction as shown next.

Objective 2 Convert mixed numbers to improper fractions. To convert the mixed number \(4\frac{3}{2}\) to an improper fraction, first multiply the denominator of the fraction part (in this case, 8) and the whole number part (in this case, 4). This gives \(4 \times 8 = 32 (4 = \frac{32}{8})\). Then add the product (32) to the numerator (in this case, 5). This gives \(32 + 5 = 37\). This sum is the numerator of the new improper fraction. The denominator stays the same.

\[
\frac{45}{8} = \frac{37}{8} \quad (4 \times 8) + 5
\]

The opening in the kitchen cabinet to the left is for a microwave oven and measures \(25\frac{3}{4}\) inches wide by \(16\frac{1}{2}\) inches high. These are both mixed numbers. The thickness of the oak wood used for the cabinet is \(\frac{1}{8}\) inch, which is a proper fraction.

The width and height of the opening in the cabinet shown are both expressed as mixed numbers. Convert these mixed numbers to improper fractions.

(a) \(25\frac{3}{4}\)  (b) \(16\frac{1}{2}\)
SOLUTION

(a) First multiply 4 (the denominator) by 25 (the whole number), and then add 3 (the numerator). This gives \((4 \times 25) + 3 = 100 + 3 = 103\). The parentheses are used to show that 4 and 25 are multiplied first.

\[
25 \frac{3}{4} = \frac{103}{4} = (4 \times 25) + 3
\]

(b) \(16 \frac{1}{2} = \frac{(2 \times 16) + 1}{2} = \frac{33}{2}\)

Objective 3] Convert improper fractions to mixed numbers. To convert an improper fraction to a mixed number, divide the numerator of the improper fraction by the denominator. The quotient is the whole-number part of the mixed number, and the remainder is used as the numerator of the fraction part. The denominator stays the same. For example, convert \(\frac{17}{5}\) to a mixed number by dividing 17 by 5.

\[
\frac{17}{5} = 3 \frac{2}{5}
\]

The whole-number part is the quotient 3. The remainder 2 is used as the numerator of the fraction part. Keep 5 as the denominator.

EXAMPLE 2

Converting Improper Fractions to Mixed Numbers

(a) \(\frac{27}{4}\) (b) \(\frac{29}{8}\) (c) \(\frac{42}{7}\)

SOLUTION

(a) Convert \(\frac{27}{4}\) to a mixed number by dividing 27 by 4.

\[
4)27\underline{\underline{27}} = 6 \frac{3}{4}
\]

The whole-number part of the mixed number is 6. The remainder 3 is used as the numerator of the fraction. Keep 4 as the denominator.

\[
\frac{27}{4} = 6 \frac{3}{4}
\]

(b) Divide 29 by 8 to convert \(\frac{29}{8}\) to a mixed number.

\[
8)29\underline{\underline{29}} = 3 \frac{5}{8}
\]

(c) Divide 42 by 7 to convert \(\frac{42}{7}\) to a mixed number.

\[
7)42\underline{\underline{42}} = 6 \frac{0}{7}
\]
QUICK TIP A proper fraction has a value that is less than 1, while an improper fraction has a value that is greater than or equal to 1.

Objective 4 Write a fraction in lowest terms. If both the numerator and denominator of a fraction cannot be divided without remainder by any number other than 1, then the fraction is in lowest terms. For example, 2 and 3 cannot be divided without remainder by any number other than 1, so the fraction $\frac{2}{3}$ is in lowest terms. In the same way, $\frac{7}{11}$, $\frac{15}{17}$, and $\frac{13}{15}$ are in lowest terms.

When both numerator and denominator can be divided without remainder by a number other than 1, the fraction is not in lowest terms. For example, both 15 and 25 may be divided by 5, so the fraction $\frac{15}{25}$ is not in lowest terms. Write $\frac{15}{25}$ in lowest terms by dividing both numerator and denominator by 5, as follows.

$$
\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}
$$

EXAMPLE 3

Writing Fractions in Lowest Terms

SOLUTION

Look for a number that can be divided into both the numerator and denominator.

(a) Both 15 and 40 can be divided by 5.

$$
\frac{15}{40} = \frac{15 \div 5}{40 \div 5} = \frac{3}{8}
$$

(b) Divide by 3.

$$
\frac{33}{39} = \frac{33 \div 3}{39 \div 3} = \frac{11}{13}
$$

Objective 5 Use the rules for divisibility. It is sometimes difficult to tell which numbers will divide evenly into another number. The following rules can sometimes help.

Rules for Divisibility

A number can be evenly divided by

2 if the last digit is an even number, such as 0, 2, 4, 6, or 8
3 if the sum of the digits is divisible by 3
4 if the last two digits are divisible by 4
5 if the last digit is 0 or 5
6 if the number is even and the sum of the digits is divisible by 3
8 if the last three digits are divisible by 8
9 if the sum of all the digits is divisible by 9
10 if the last digit is 0

EXAMPLE 4

Using the Divisibility Rules

Determine whether the following statements are true.

(a) 3,746,892 is divisible by 4.
(b) 15,974,802 is divisible by 9.

SOLUTION

(a) The number 3,746,892 is divisible by 4, since the last two digits form a number divisible by 4.

$$
3,746,892
$$

92 is divisible by 4.
(b) See if 15,974,802 is divisible by 9 by adding the digits of the number.

\[ 1 + 5 + 9 + 7 + 4 + 8 + 0 + 2 = 36 \]

Since 36 is divisible by 9, the given number is divisible by 9.

**QUICK TIP** Testing for divisibility by adding the digits works only for 3 and 9.

The rules for divisibility only help determine whether a number is evenly divisible by another number. They cannot be used to find the result. You must carry out the division to find the quotient.
2.2 Addition and Subtraction of Fractions

Objectives
1. Add and subtract like fractions.
2. Find the least common denominator.
3. Add and subtract unlike fractions.
4. Rewrite fractions with a common denominator.

CASE POINT
Sarah Bryn must use fractions on a daily basis as she works with contractors and homeowners at The Home Depot. The measurements of cabinets, trim pieces, and room sizes never seem to be even numbers of inches—they always have fractions of an inch.

Objective 1 Add and subtract like fractions. Fractions with the same denominator are called like fractions. Such fractions have a common denominator. For example, \( \frac{1}{4} \) and \( \frac{2}{4} \) are like fractions with a common denominator of 4, while \( \frac{1}{2} \) and \( \frac{3}{4} \) are not like fractions. Add or subtract like fractions by adding or subtracting the numerators, and then place the result over the common denominator.

EXAMPLE 1

Adding and Subtracting Like Fractions

Add or subtract.
(a) \( \frac{3}{4} + \frac{1}{4} + \frac{5}{4} \)  
(b) \( \frac{11}{15} - \frac{4}{15} \)

SOLUTION
The fractions in both parts of this example are like fractions. Add or subtract the numerators and place the result over the common denominator.

(a) \( \frac{3}{4} + \frac{1}{4} + \frac{5}{4} = \frac{3 + 1 + 5}{4} = \frac{9}{4} = 2 \frac{1}{4} \)

(b) \( \frac{11}{15} - \frac{4}{15} = \frac{11 - 4}{15} = \frac{7}{15} \)

Objective 2 Find the least common denominator. Fractions with different denominators, such as \( \frac{1}{2} \) and \( \frac{3}{4} \), are unlike fractions. Add or subtract unlike fractions by first writing the fractions with a common denominator. The least common denominator (LCD) for two or more fractions is the smallest whole number that can be divided, without remainder, by all the denominators of the fractions. For example, the LCD of the fractions \( \frac{1}{2} \), \( \frac{3}{4} \), and \( \frac{1}{3} \) is 12, since 12 is the smallest number that can be divided by 4, 6, and 2.

Notice that the fractions shown in the following shelf-end base drawing are like fractions, \( \frac{23}{15} \), \( \frac{10}{7} \), and \( \frac{11}{12} \). However, in the drawing of the shelf-end peninsula base, the fractions are unlike fractions, \( \frac{22}{16} \), \( \frac{11}{3} \), and \( \frac{11}{8} \).
There are two methods of finding the least common denominator.

**Inspection.** With small denominators, it may be possible to find the least common denominator by inspection. For example, the LCD for $\frac{1}{3}$ and $\frac{1}{5}$ is 15, the smallest number that can be divided evenly by both 3 and 5.

**Method of prime numbers.** If the LCD cannot be found by inspection, use the method of prime numbers, as explained in the next example.

A *prime number* is a number that can be divided without remainder by exactly two distinct numbers: itself and 1. Prime numbers are 2, 3, 5, 7, 11, 13, 17, and so on. (1 is not prime, because it can be divided evenly by only one number: the number 1.)

### QUICK TIP
All prime numbers other than 2 are odd numbers. Not all odd numbers, however, are prime numbers. For example, 27 is the product of 3 and 9.

#### Example 2
Finding the Least Common Denominator

Use the method of prime numbers to find the least common denominator for $\frac{2}{12}$, $\frac{3}{18}$, and $\frac{5}{20}$.

**SOLUTION**
First write the three denominators: 12 18 20.

Begin by trying to divide the three denominators by the first prime number, 2. Write each quotient directly above the given denominator as follows.

\[
\begin{array}{ccc}
6 & 9 & 10 \\
\\n2 & 12 & 18 & 20 \\
\end{array}
\]

This way of writing the division is just a handy way of writing the separate problems $\frac{6}{2}$, $\frac{9}{12}$, and $\frac{10}{18}$. Two of the new quotients, 6 and 10, can still be divided by 2, so perform the division again. Since 9 cannot be divided evenly by 2, just bring up the 9.

\[
\begin{array}{ccc}
3 & 9 & 5 \\
\\n2 & 12 & 18 & 20 \\
\end{array}
\]

None of the new quotients in the top row can be divided by 2, so try the next prime number, 3. The numbers 3 and 9 can be divided by 3, and one of the new quotients can still be divided by 3, so the division is performed again.

\[
\begin{array}{ccc}
1 & 1 & 5 \\
\\n3 & 1 & 3 & 5 \\
\\n3 & 3 & 9 & 5 \\
\\n2 & 6 & 9 & 10 \\
\\n2 & 12 & 18 & 20 \\
\end{array}
\]

Since none of the new quotients in the top row can be divided by 3, try the next prime number, 5. The number 5 can be used only once, as shown.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
\\n5 & 1 & 1 & 5 \\
\\n3 & 1 & 3 & 5 \\
\\n3 & 3 & 9 & 5 \\
\\n2 & 6 & 9 & 10 \\
\\n2 & 12 & 18 & 20 \\
\end{array}
\]

Now that the top row contains only 1's, find the least common denominator by multiplying the prime numbers in the left column.

The least common denominator is $2 \times 2 \times 3 \times 3 \times 5 = 180$. 
2.2 ADDITION AND SUBTRACTION OF FRACTIONS

**Example 3**

Finding the Least Common Denominator

Find the least common denominator for $\frac{1}{5}$, $\frac{3}{11}$, and $\frac{6}{25}$.

**Solution**

Write the denominators in a row and use the method of prime numbers.

\[
\begin{array}{ccc}
5 & 11 & 15 \\
3 & 11 & 3 & 5 \\
2 & 2 & 3 & 5 \\
2 & 4 & 6 & 5 \\
2 & 8 & 12 & 10
\end{array}
\]

Start here →

The least common denominator is $2 \times 2 \times 3 \times 5 = 120$.

**Quick Tip**

Sometimes it is tempting to use a number that is not prime when solving for the least common denominator. This should be avoided because the result is often something different from the least common denominator.

Unlike fractions may be added or subtracted using the following steps.

**Adding or Subtracting Unlike Fractions**

- **Step 1** Find the least common denominator (LCD).
- **Step 2** Rewrite the unlike fractions as like fractions having the least common denominator.
- **Step 3** Add or subtract numerators, placing answers over the LCD and reducing to lowest terms.

**Objective 3** Add and subtract unlike fractions. To add or subtract unlike fractions, rewrite the fractions with a common denominator. Since Example 2 shows that 180 is the least common denominator for $\frac{2}{15}$, $\frac{3}{18}$, and $\frac{4}{20}$, these three fractions can be added if each fraction is first written with a denominator of 180.

**Step 1**

\[
\frac{5}{12} = \frac{15}{180} \quad \frac{7}{18} = \frac{18}{180} \quad \frac{11}{20} = \frac{99}{180}
\]

**Objective 4** Rewrite fractions with a common denominator. To rewrite the preceding fractions with a common denominator, first divide each denominator from the original fractions into the common denominator.

**Step 2**

\[
\frac{15}{12/180} = \frac{10}{18/180} = \frac{9}{20/180}
\]

Next multiply each quotient by the original numerator.

\[
15 \times 5 = 75 \quad 10 \times 7 = 70 \quad 9 \times 11 = 99
\]

Now, rewrite the fractions.

\[
\frac{5}{12} = \frac{75}{180} \quad \frac{7}{18} = \frac{70}{180} \quad \frac{11}{20} = \frac{99}{180}
\]
Now add the fractions.

Step 3
\[
\frac{5}{12} + \frac{7}{18} + \frac{11}{20} = \frac{75}{180} + \frac{70}{180} + \frac{99}{180} = \frac{244}{180} = \frac{64}{180} = \frac{16}{45}
\]
Write the answer as a mixed number with the fraction in lowest terms.

**EXAMPLE 4**

Adding and Subtracting Unlike Fractions

Add or subtract.

(a) \(\frac{3}{4} + \frac{1}{2} + \frac{5}{8}\)  
(b) \(\frac{9}{10} - \frac{3}{8}\)

**SOLUTION**

(a) Inspection shows that the least common denominator is 8. Rewrite the fractions so they each have a denominator of 8. Then add.

\[
\frac{3}{4} + \frac{1}{2} + \frac{5}{8} = \frac{6}{8} + \frac{4}{8} + \frac{5}{8} = \frac{6 + 4 + 5}{8} = \frac{15}{8} = 1\frac{7}{8}
\]

(b) The least common denominator is 40. Rewrite the fractions so they each have a denominator of 40. Then subtract.

\[
\frac{9}{10} - \frac{3}{8} = \frac{36}{40} - \frac{15}{40} = \frac{21}{40}
\]

Fractions can also be added or subtracted vertically, as shown in the next example.

**EXAMPLE 5**

Adding and Subtracting Unlike Fractions

Add or subtract.

(a) \(\frac{2}{9} + \frac{3}{4}\)  
(b) \(\frac{11}{16} + \frac{7}{12}\)  
(c) \(\frac{7}{8} - \frac{5}{12}\)

**SOLUTION**

First rewrite the fractions with a least common denominator.

(a) \(\frac{2}{9} = \frac{8}{36}\)  
(b) \(\frac{11}{16} = \frac{33}{48}\)  
(c) \(\frac{7}{8} = \frac{21}{24}\)

\[
+ \frac{3}{4} = \frac{27}{36} + \frac{7}{12} = \frac{28}{48} + \frac{5}{12} = \frac{13}{48}
\]

All calculator solutions are shown using a basic calculator. The calculator solution to Example 5(b) uses the fraction key on the calculator.

\[
\frac{11}{16} + \frac{7}{12} - \frac{1}{3}
\]

*Note: Refer to Appendix C for calculator basics.*
2.3 Addition and Subtraction of Mixed Numbers

Objectives
1. Add mixed numbers.
2. Add with carrying.
3. Subtract mixed numbers.
4. Subtract with borrowing.

CASE POINT
Total customer satisfaction is important to Sarah Bryn and The Home Depot. Complete accuracy is just as important in the small jobs in hardware and trim as it is in the large jobs in cabinets and installation. To achieve this accuracy, Bryn knows that mixed numbers must be added and subtracted carefully and that all calculations must then be checked to make sure they are correct.

Objective 1 Add mixed numbers. To add mixed numbers, first add the fractions. Then add the whole numbers and combine the two answers. For example, add $16\frac{1}{8}$ and $5\frac{3}{8}$ as shown.

\[
\begin{align*}
\text{sum of fractions} & = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} \\
& = \frac{1}{2} \\
\text{sum of whole numbers} & = 16 + 5 = 21
\end{align*}
\]

Write $\frac{4}{8}$ in lowest terms as $\frac{1}{2}$, so that $16\frac{1}{8} + 5\frac{3}{8} = 21\frac{1}{2}$.
To add mixed numbers, change the mixed numbers, if necessary, so that the fraction parts have a common denominator.

Add $9\frac{1}{3}$ and $6\frac{1}{2}$.

SOLUTION
Inspection shows that 12 is the least common denominator. Write $9\frac{1}{3}$ as $9\frac{4}{12}$, and write $6\frac{1}{2}$ as $6\frac{6}{12}$. Then add. The work can be organized as follows.

\[
\begin{align*}
9\frac{2}{3} & = 9\frac{8}{12} \\
+ 6\frac{1}{4} & = 6\frac{3}{12} \\
\hline \\
& = 15\frac{11}{12}
\end{align*}
\]

Objective 2 Add with carrying. If the sum of the fraction parts of mixed numbers is greater than 1, carry the excess from the fraction part to the whole number part.

A rubber gasket must extend around all four edges (perimeter) of the dishwasher door panel shown below before it is installed. Find the length of gasket material needed. Add $34\frac{1}{2}$ inches, $23\frac{3}{4}$ inches, $34\frac{1}{2}$ inches, and $25\frac{1}{4}$ inches.

![Dishwasher door panel diagram]
CHAPTER 2 FRACTIONS

SOLUTION
\[
\begin{align*}
34 \frac{1}{2} &= 34 \frac{2}{4} \\
23 \frac{3}{4} &= 23 \frac{3}{4} \\
34 \frac{1}{2} &= 34 \frac{2}{4} \\
+23 \frac{3}{4} &= 23 \frac{3}{4} \\
\hline
114 \frac{10}{4} &= 114 + \frac{10}{4} = 114 + 2\frac{2}{4} = 116\frac{2}{4} = 116\frac{1}{2} \text{ inches length of gasket needed}
\end{align*}
\]

QUICK TIP
When adding mixed numbers, first add the fraction parts. Then add the whole-number parts. Finally, combine the two answers.

Objective 3 Subtract mixed numbers. To subtract two mixed numbers, change the mixed numbers, if necessary, so that the fraction parts have a common denominator. Then subtract the fraction parts and the whole-number parts separately. For example, subtract \(3\frac{5}{8}\) from \(8\frac{3}{4}\) by first finding that the least common denominator is 24. Then rewrite the problem as shown.

\[
\begin{align*}
8\frac{5}{8} - 3\frac{1}{12} &= 8\frac{5}{8} - 3\frac{1}{12} \\
\text{as a common denominator} &= \frac{15}{24} - \frac{3}{24} \\
\text{Subtract the fraction parts} &= \frac{12}{24} \\
\text{and subtract the whole-number parts.} &= 5\frac{13}{24} \\
\text{Subtract fractions.} &= 5\frac{13}{24} \\
\text{Subtract whole numbers.} &= 5
\end{align*}
\]

Objective 4 Subtract with borrowing. The following example shows how to subtract when borrowing is needed.

(a) Subtract \(6\frac{3}{4}\) from \(10\frac{1}{4}\).

(b) Subtract \(15\frac{1}{2}\) from 41.

SOLUTION
Start by rewriting each problem with a common denominator.

(a) \(10\frac{1}{4} = 10\frac{1}{4}\)

Subtracting \(\frac{3}{4}\) from \(\frac{1}{4}\) requires borrowing from the whole number 10.

\[
10\frac{1}{4} = 9 + 1 + \frac{1}{4}
\]

Rewrite the problem as shown.
Check by adding \(3\frac{3}{4}\) and \(6\frac{3}{4}\).
The answer should be \(10\frac{1}{4}\).

\[
\begin{align*}
10\frac{1}{4} &= 9\frac{3}{4} \\
-6\frac{3}{4} &= 6\frac{3}{4} \\
\hline
3\frac{1}{4} &= \frac{1}{4}
\end{align*}
\]

(b) \(41\frac{1}{4} = 41\frac{1}{4}\)

To subtract the fraction \(\frac{1}{2}\) requires borrowing 1 whole unit from 41.

\[
41 = 40 + 1 = 40 + \frac{1}{2} = 40\frac{1}{2} = 1 = \frac{1}{2}
\]

Rewrite the problem as shown.
Check by adding \(25\frac{1}{2}\) and \(15\frac{1}{2}\).
The answer should be 41.

\[
\begin{align*}
41 &= 40\frac{1}{2} \\
-15\frac{1}{2} &= 15\frac{1}{2} \\
\hline
25\frac{1}{2}
\end{align*}
\]

The calculator solution to Example 3(a) uses the fraction key.

\[
10 + \frac{3}{4} + 8 + 6 = \frac{3}{4} + 4 + \frac{3}{4}
\]

Note: Refer to Appendix C for calculator basics.
2.4 Multiplication and Division of Fractions

Objectives
1. Multiply proper fractions.
2. Use cancellation.
3. Multiply mixed numbers.
4. Divide fractions.
5. Divide mixed numbers.
6. Multiply or divide by whole numbers.

CASE POINT
Most of the cabinets sold by The Home Depot are standard size units and modules that can be combined to satisfy varied applications and room sizes. However, all too often, Sarah Bryn finds that various components and trim pieces must be custom sized. In order to custom size items, she must multiply and divide fractions.

Objective 1 Multiply proper fractions. To multiply two fractions, first multiply the numerators to form a new numerator and then multiply the denominators to form a new denominator. Write the answer in lowest terms if necessary. For example, multiply \( \frac{5}{8} \) and \( \frac{3}{4} \) by first multiplying the numerators and then the denominators. This gives

\[
\frac{2}{3} \times \frac{5}{8} = \frac{2 \times 5}{3 \times 8} = \frac{10}{24} = \frac{5}{12} \quad \text{(in lowest terms)}
\]

Multiply numerators. Multiply denominators.

Objective 2 Use cancellation. This problem can be simplified by cancellation, a modification of the method of writing fractions in lowest terms. For example, find the product of \( \frac{1}{3} \) and \( \frac{5}{12} \) by cancelling as follows.

\[
\frac{1}{3} \times \frac{5}{12} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}
\]

Divide 2 into both 2 and 8. Then multiply the numerators and, finally, multiply the denominators.

EXAMPLE 1

Multiplying Common Fractions

(a) \( \frac{8}{15} \times \frac{5}{12} \) \hspace{1cm} (b) \( \frac{35}{12} \times \frac{32}{25} \)

SOLUTION

Use cancellation in both of these problems.

(a) \( \frac{2}{3} \times \frac{5}{12} = \frac{2 \times 1}{3 \times 3} = \frac{2}{9} \)

Divide 4 into both 8 and 12. Divide 3 into both 3 and 15.

(b) \( \frac{25}{12} \times \frac{8}{25} = \frac{7 \times 8}{3 \times 5} = \frac{56}{15} = \frac{3}{15} \)

Divide 4 into both 12 and 32. Divide 5 into both 5 and 25.

QUICK TIP When cancelling, be certain that a numerator and a denominator are both divided by the same number.
Objective 1. Multiply mixed numbers. To multiply mixed numbers, change the mixed numbers to improper fractions, use cancellation, and then multiply them. For example, multiply $6\frac{1}{4}$ and $2\frac{2}{3}$ as follows.

$$6\frac{1}{4} \times 2\frac{2}{3} = \frac{25}{4} \times \frac{8}{3} = \frac{25}{4} \times \frac{2}{3} = \frac{50}{3} = 16\frac{2}{3}$$

**QUICK TIP** Mixed numbers must always be changed to improper fractions before multiplying. When multiplying by mixed numbers, do not multiply whole numbers by whole numbers and fractions by fractions.

**EXAMPLE 2**

**Multiplying Mixed Numbers**

Multiply.

(a) $3\frac{3}{4} \times 8\frac{2}{3}$

(b) $1\frac{3}{5} \times 3\frac{1}{3} \times 1\frac{3}{4}$

**SOLUTION**

(a) $\frac{15}{4} \times \frac{26}{3} = \frac{5 \times 13}{2 \times 1} = \frac{65}{2} = 32\frac{1}{2}$

(b) $\frac{8}{5} \times \frac{10}{3} \times \frac{2}{4} = \frac{2 \times 2 \times 7}{1 \times 3 \times 1} = \frac{28}{3} = 9\frac{1}{3}$

The calculator solution to Example 2(b) uses the fraction key.

```
1 a/b 3 a/b 5 a/b 2 a/b 3 a/b 1 a/b 3 a/b 1 a/b 3 a/b 4 a/b 9 1
```

*Note: Refer to Appendix C for calculator basics.*

The recipe shown next is easy to follow using proper measuring cups and spoons. Sometimes you may want to double or triple a recipe or perhaps, cooking for a small group, you need to cut the recipe in half. To double the recipe, multiply each ingredient by 2. To triple the recipe, multiply by 3. To halve the recipe you’ll need to divide by 2.

**Chocolate/Oat-Chip Cookies**

1 cup (2 sticks) margarine or butter, softened

1$\frac{1}{2}$ cups firmly packed brown sugar

$\frac{1}{2}$ cup granulated sugar

2 eggs

2 tablespoons milk

2 teaspoons vanilla

1$\frac{1}{2}$ cups all-purpose flour

1 teaspoon baking soda

Heat oven to 375°F. Beat margarine and sugars until creamy.

Add eggs, milk, and vanilla; beat well.

Add combined flour, baking soda, and salt; mix well. Stir in oats, chocolate morsels, and nuts; mix well.

Drop by rounded measuring tablespoons onto ungreased cookie sheet.

Bake 9 to 10 minutes for a chewy cookie or 12 to 13 minutes for a crisp cookie.

Cool 1 minute on cookie sheet; remove to wire rack. Cool completely

**MAKES ABOUT 5 DOZEN**
EXAMPLE 3

Multiplying Mixed Numbers by a Whole Number

(a) Find the amount of uncooked oats needed if the preceding recipe for chocolate/ Nutri- chip cookies is doubled (multiplied by 2).

(b) How many cups of all-purpose flour are needed when the recipe is tripled (multiplied by 3)?

SOLUTION

(a) \(2 \frac{1}{2} \times 2 = \frac{5}{2} \times \frac{1}{1} = \frac{5 \times 1}{2 \times 1} = \frac{5}{2} = 5 \text{ cups}\)

(b) \(1 \frac{3}{4} \times 3 = \frac{7}{4} \times \frac{3}{1} = \frac{7 \times 3}{4} = \frac{21}{4} = 5 \frac{1}{4} \text{ cups}\)

Objective 41 Divide fractions. To divide two fractions, invert the second fraction (divisor) and then multiply the first fraction by the inverted second fraction. (Invert a fraction by exchanging the numerator and the denominator.)

For example, divide \(\frac{3}{8}\) by \(\frac{7}{12}\) by inverting the second fraction and then multiplying.

\[
\frac{3}{8} \div \frac{7}{12} = \frac{3}{8} \times \frac{12}{7} = \frac{3 \times 12}{8 \times 7} = \frac{36}{56} = \frac{9}{14}
\]

QUICK TIP

Only the second fraction (divisor) is inverted when dividing by a fraction. Cancellation is done only after inverting.

EXAMPLE 4

Dividing Common Fractions

Divide.

(a) \(\frac{7}{8} \div \frac{1}{4}\)  
(b) \(\frac{25}{36} \div \frac{15}{18}\)

SOLUTION

Invert the second fraction and then multiply.

(a) \(\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} = \frac{7 \times 4}{8 \times 1} = \frac{28}{8} = 3 \frac{1}{2}\)

(b) \(\frac{25}{36} \div \frac{15}{18} = \frac{25}{36} \times \frac{18}{15} = \frac{25 \times 18}{36 \times 15} = \frac{5 \times 1}{2 \times 3} = \frac{5}{6}\)

Objective 51 Divide mixed numbers. To divide mixed numbers, first change all mixed numbers to improper fractions, invert the second fraction, use cancellation, and multiply.

\[
3 \frac{5}{9} \div 2 \frac{2}{5} = \frac{32}{9} \div \frac{12}{5} = \frac{32}{9} \times \frac{5}{12} = \frac{8 \times 5}{9 \times 3} = \frac{40}{27} = 1 \frac{13}{27}
\]

Objective 61 Multiply or divide by whole numbers. To multiply or divide a fraction by a whole number, write the whole number as a fraction over 1.

Multiply: \(\frac{3}{4} \times 16 = \frac{3}{4} \times \frac{16}{1} = \frac{15}{4} \times \frac{16}{1} = \frac{15 \times 16}{4 \times 1} = 15 \times 4 = 60\)

Divide: \(\frac{2 \frac{2}{5}}{3} = \frac{12}{5} \div \frac{3}{1} = \frac{12}{5} \times \frac{1}{3} = \frac{4 \times 1}{5 \times 1} = \frac{4}{5}\)
Mills Pride manufactures cabinets for kitchens and baths. The specifications for base-end panels are shown in the diagram. The lumber used is $\frac{3}{4}$ inch thick and is cut down from 24 inches to a $23\frac{3}{4}$-inch width. The panel is then cut to a height of $34\frac{1}{2}$ inches. The materials used in the manufacture of cabinets, solid oak in this case, are very expensive. Every precaution is taken to ensure a minimum of wasted material.

**EXAMPLE 5**

**Multiplying a Whole Number by a Mixed Number**

A cabinet maker will need 80 base-end panels to complete a job. If each panel is $34\frac{1}{2}$ inches long, how many inches of oak material are needed?

**SOLUTION**

Multiply the number of panels needed by the length of each panel: $34\frac{1}{2}$, or $\frac{69}{2}$.

$$80 \times \frac{69}{2} = \frac{80}{1} \times \frac{69}{2} = \frac{40 \times 69}{1 \times 1} = 2760$$

The length of material needed by the cabinet maker is 2760 inches.

**QUICK TIP** It is often best to change a fraction or mixed number to a decimal number. This procedure is discussed in *Section 2.5*.

**EXAMPLE 6**

**Dividing a Whole Number by a Mixed Number**

To complete a custom-designed cabinet, oak trim pieces must be cut exactly $2\frac{1}{4}$ inches long so that they can be used as dividers in a spice rack. Find the number of pieces that can be cut from a piece of oak that is 54 inches in length.

**SOLUTION**

To divide the length of the piece of oak by $2\frac{1}{4}$, or $\frac{9}{4}$, invert and then multiply.

$$54 \div \frac{9}{4} = 54 \times \frac{4}{9} = \frac{54 \times 4}{1 \times 1} = \frac{24}{1} = 24$$

The number of trim pieces that can be cut from the oak stock is 24.
2.5 Converting Decimals to Fractions and Fractions to Decimals

Objectives
1. Convert decimals to fractions.
2. Convert fractions to decimals.
3. Know common decimal equivalents.

Objective 1. **Convert decimals to fractions.** A common method of converting a decimal to a fraction is by thinking of the decimal as being written in words, as in the preceding chapter. For example, think of .47 as “forty-seven hundredths.” Then write this in fraction form as

\[ .47 = \frac{47}{100} \]

In the same way, .3, read as “three tenths,” is written in fraction form as

\[ .3 = \frac{3}{10} \]

Also, .963, read “nine hundred sixty-three thousandths,” is written in fraction form as

\[ .963 = \frac{963}{1000} \]

Another method of converting a decimal to a fraction is by first removing the decimal point. The remaining number is the numerator of the fraction. The denominator of the fraction is 1 followed by as many zeros as there were digits to the right of the decimal point in the original number.

**EXAMPLE 1**

**Converting Decimals to Fractions**

(a) .3  
(b) .98  
(c) .654

**SOLUTION**

(a) There is one digit following the decimal point in .3. Make a fraction with 3 as the numerator. For the denominator, use 10, which is 1 followed by one zero.

\[ .3 = \frac{3}{10} \]

This fraction is in lowest terms.

(b) There are two digits following the decimal point in .98. Make a fraction with 98 as the numerator and 100 as the denominator.

\[ .98 = \frac{98}{100} = \frac{49}{50} \] (lowest terms)

(c) There are three digits following the decimal point in .654.

\[ .654 = \frac{654}{1000} = \frac{327}{500} \] (lowest terms)

Objective 2. **Convert fractions to decimals.** Convert a fraction to a decimal by dividing the numerator of the fraction by the denominator. Place a decimal point after the numerator and attach one zero at a time to the right of the decimal point as the division is performed. Keep going until the division produces a remainder of zero or until the desired degree of accuracy is reached.
EXAMPLE 2
Converting Fractions to Decimals

Decimal Equivalents

\[
\begin{align*}
\frac{1}{16} &= .0625 \\
\frac{1}{8} &= .125 \\
\frac{1}{4} &= .1111 \text{ (rounded)} \\
\frac{1}{2} &= .5 \\
\frac{5}{8} &= .625 \\
\frac{3}{4} &= .6666 \text{ (rounded)} \\
\frac{7}{8} &= .875 \\
\frac{9}{16} &= .5625 \\
\frac{11}{16} &= .6875 \\
\frac{1}{16} &= .0625 \\
\frac{3}{16} &= .375 \\
\frac{7}{16} &= .4375 \\
\frac{11}{16} &= .6875 \\
\frac{1}{4} &= .25 \\
\frac{3}{10} &= .3 \\
\frac{7}{10} &= .7 \\
\frac{1}{25} &= .04 \\
\frac{1}{50} &= .02 \\
\frac{1}{100} &= .01 \\
\frac{1}{1000} &= .001
\end{align*}
\]

Convert the following fractions to decimals.

(a) \(\frac{1}{8}\) \hspace{1cm} (b) \(\frac{2}{3}\)

SOLUTION

(a) Convert \(\frac{1}{8}\) to a decimal by dividing 1 by 8.

\[
8 \overline{1.0}
\]

Since 8 will not divide into 1, place a 0 to the right of the decimal point. Now 8 goes into 10 once, with a remainder of 2.

\[
\begin{align*}
8 &\overset{.1}{\overline{1.0}} \\
8 &\overset{.0}{\overline{2}}
\end{align*}
\]

Be sure to move the decimal point up.

Continue placing zeros to the right of the decimal point and continue dividing. The division now gives a remainder of 0.

\[
\begin{align*}
\frac{1}{8} &= .125 \\
8 \overline{.000} &\text{ Keep attaching zeros.}
\end{align*}
\]

Therefore, \(\frac{1}{8} = .125\).

(b) Divide 2 by 3.

\[
\begin{align*}
0.6666 &\text{ Keep attaching zeros.} \\
\frac{2}{3} &= 0.6666 \\
1.8 &\text{ (rounded)} \\
20 &\text{ (rounded)} \quad \frac{1}{3} \approx .333
\end{align*}
\]

This division results in a repeating decimal and is often written as \(0.\overline{6}\), \(0.66\), or \(0.666\). Rounded to the nearest thousandth, \(\frac{1}{3} = .667\).

The calculator solution to this example is

\[
2 \div 3 \approx 0.66666667
\]

Note: Refer to Appendix C for calculator basics.

Objective Know common decimal equivalents. Some of the more common decimal equivalents of fractions are listed in the margin. These decimals appear from least to greatest value and are rounded to the nearest ten-thousandth. Sometimes decimals must be carried out further to give greater accuracy, while at other times they are not carried out as far and are rounded sooner.