DECIMALS AND PERCENTS

Ordering by tens

More than incidental use of decimals can be found in the mathematics of ancient China and medieval Arabia. However, it was in the *Compendio de lo abaco*, published in the year Columbus journeyed to America, that the use of the decimal point appeared to denote the division of an integer by powers of 10. The earliest systematic treatment of decimals was given by the Flemish mathematician Simon Stevin in his arithmetic text of 1595 entitled *La Disme*. He developed a notation quite similar to ours; each digit is followed by a circled number that indicates the number of places that the digit is to the right of the units place. For example, 3.1 would be written as 3 1 1 4 2.

In this chapter we discuss numbers that can be written with decimal points or as percents. Our money is expressed in decimals, and interest is expressed in percents. Everyone has experience with money and with loans. You will find that you are already familiar with many of the operations involving decimals and percents.

3.1 DECIMALS

Place Values

Our number system is called a *decimal system*. The prefix *deci* means ten. In the decimal system, we use ten symbols—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—and we count in groups of ten: 10, 20, 30, 40, and so on. (For the Laplanders, with one hand hidden in a mitten, 5 was the central number or base of their number system. The Mayans, in Mexico, used 20, possibly because they were barefoot and could use both fingers and toes as aids to counting.) In computer science, the base 2 is used because in an off/on system there are only two possibilities: a switch is either on or off, there is or is not an electric signal, etc.

As an example of place values in the decimal system, look at the number 222. Starting at the left, the first 2 has a value of 200 (2 \( \times \) 100), the second has a value of 20 (2 \( \times \) 10), and the last 2 has a value of 2 (2 \( \times \) 1). In each place we can have
any of our ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). From each digit's place in the number we are considering, we know its value (e.g., in 34, 3 represents 30 and 4 represents 4).

Consider the number 5. If you move the 5 one place to the left, you get 50; you have multiplied 5 by 10. Move 5 two places to the left and you get 500; you have multiplied 5 by 100 or $10 \times 10$. (This is, by the way, how you enter numbers when you use a calculator or when you deposit money into a cash machine at the bank: you press one number key at a time, and the digits you have already keyed in move one place to the left each time.)

Now, instead of moving 5 to the left, move it one place to the right. What happens?

5 becomes .5

The rightmost place value for a whole number is the ones (units) place. We can mark that with a period called the decimal point and then continue with new places to the right of the decimal point. Thus the whole number 5 could be written as 5., and when we move the digit 5 one place to the right we have .5 as the new number.

**EXAMPLE**

Line 1 indicates how one dollar ($1.00) would be entered in this table.

<table>
<thead>
<tr>
<th>Ones Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0 0</td>
</tr>
<tr>
<td>2. 0</td>
</tr>
<tr>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
</tr>
</tbody>
</table>

(a) Show where 10 dollars belongs in line 2.
(b) Show where 100 cents belongs in line 3.
(c) Show where 10 cents belongs in line 4.
(d) Show where 1 cent belongs in line 5.

**Solution** Since 100 cents = $1.00
10 cents = $0.10
1 cent = $0.01

we get the following table:

<table>
<thead>
<tr>
<th>hundreds</th>
<th>tens</th>
<th>ONES</th>
<th>tenths</th>
<th>hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Line 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Line 3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Line 4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Line 5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The decimal point shows where the ones (or units) place is. Notice that the decimal numbers are symmetrical around the ones place: one step to the left of 1 gives us the place value of ten, while one step to the right of 1 has the
value of one tenth. Two steps to the left of 1 is the hundreds place; two steps to the right is the hundredths place.
Earlier, when we moved 5 one place to the right, 5 moved to the tenths place.
We call its new value "five tenths." (In daily language we say "point five.")
To go from 100 to 10, we divide by 10.
To go from 10 to 1, we divide by 10.
To go from 1 to the tenths place, we also divide by 10. That place’s value is 1/10, or one tenth. We can write one tenth as .1 or 0.1.

**DEFINITION**

Place values:
Ones, Tenths Hundredths Thousandths Ten-thousandths...
(see Figure 3.1 below)

**Reading Decimals**
The number 1.234 has 1 in the units place, 2 in the tenths place, 3 in the hundredths place, and 4 in the thousandths place. We read the number as "one and two hundred thirty-four thousandths." Notice that we say "and" to indicate the decimal point. We always use the last place as the name for the decimal part. In daily life, however, we usually say, "one point two three four."
12.0578 is read "twelve and five hundred seventy-eight ten-thousandths." 0.1 and .1 are both read as "one tenth"; in such a case we ignore the digit zero in the ones place. However, we might also say "zero point one."

**EXAMPLE**
Read: (a) 0.678, (b) 600.078, (c) 60.0078, (d) 60007.8

**Solution**
(a) six hundred seventy-eight thousandths
(b) six hundred and seventy-eight thousandths
(c) sixty and seventy-eight ten-thousandths
(d) sixty thousand seven and eight tenths
Remember to read the decimal point as "and"—(a) and (b) illustrate what a difference that makes.

![Figure 3.1 Place values.](image)
PART I  ARITHMETIC

EXAMPLE
Write the number five thousand and two hundred fifty-three ten-thousandths in mathematical symbols.

Solution  The number is 5000.0253. We must fill the empty places with zeros.

RULE

READING DECIMALS
The decimal point is read as "and." The decimals are read as whole numbers followed by the name of the rightmost place.

EXERCISE 3.1.1
1. In which place is the digit 5?
   (a) 3.056
   (b) 45.37
   (c) 10.2358
   (d) 1,053.698.23
   (e) 203.561
   (f) 1.236952

2. Write out the number as you would read it.
   (a) 98.6
   (b) 45.34
   (c) 0.678
   (d) 7.190
   (e) 10.06
   (f) 15.3829

3. Write in decimal notation.
   (a) Six and five tenths
   (b) Seven and three hundredths
   (c) Twenty-two and fifteen hundredths
   (d) Thirty and one hundred two thousandths
   (e) One hundred and six hundredths
   (f) Thirty-six ten-thousandths

Ordering Decimals
You can always tell which one of two numbers is the larger, so you can arrange any group of numbers in order. To order whole numbers or decimals we use our knowledge of place value. 20 is greater than 18 (20 > 18) because the digit in the largest place in 20 (the 2 in the tens place) is larger than the digit in the corresponding place in 18 (2 > 1).

EXAMPLE
Which is larger, 0.3 or 0.098?
CHAPTER 3  DECIMALS AND PERCENTS

Solution  Look at corresponding place values.

<table>
<thead>
<tr>
<th>tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.098</td>
</tr>
</tbody>
</table>

The largest place is the tenths, and $3 > 0$. Therefore 0.3 is larger than 0.098. We don’t have to pay attention to the hundredths or the thousandths place in this example.

EXAMPLE

Which is larger, 0.245 or 0.268?

Solution  If we compare 0.245 and 0.268, we see that in both numbers there is a 2 in the tenths place. Therefore we go to the next place to the right, the hundredths, and compare the digits. 6 is larger than 4 ($6 > 4$), so 0.268 is larger than 0.245. Two hundred sixty-eight thousandths is larger than two hundred forty-five thousandths.

EXAMPLE

Find the smallest number:
(a) 0.095, 0.1, 0.16
(b) 3, 0.333, 0.3
(c) 0.85, 0.9, 0.099

Solution
(a) 0.095
| 0.1     |
| 0.16    |

The digits in the tens place are 0 and $1.0 < 1$, so 0.095 is the smallest number.
(b) 3.000
| 0.333   |
| 0.300   |

If we fill in the empty spaces with zeros, we see clearly that 0.3 is the smallest number.
(c) 0.850
| 0.900   |
| 0.099   |

The smallest number is 0.099, because it has zero in the tens place.

RULE

ORDERING DECIMALS

Arrange the numbers in a column with the decimal points below each other. Compare the place values going from left to right.
PART I ARITHMETIC

EXERCISE 3.1.2

1. Find the largest number. (Remember that if there is no decimal point visible, it is to the right of the number: 3 is 3., 29 is 29., and so on.)
   (a) 2, 0.1, 0.9
   (b) 0.055, 0.0099, 0.2
   (c) 0.006, 0.060, 0.02
   (d) 0.95, 0.099, 0.9
   (e) 1.03, 1.1, 1.008, 1, 1.0
   (f) 2.05, 2.049, 2, 2.1, 2.13

2. Arrange in order from the smallest to the largest.
   (a) 2, 0.020, 0.2
   (b) 0.95, 0.09, 0.9
   (c) 1.2, 0.120, 12
   (d) 0.0005, 0.05, 0.005, 0.5
   (e) 0.089, 0.091, 0.19, 0.91, 0.10
   (f) 0.014, 0.019, 0.020, 0.010, 0.050

Rounding Decimals

The procedure for rounding decimals is the same as for rounding whole numbers, except that we don’t add zeros at the end. We cut off the digits we don’t need. For instance, 45.679 rounded to the nearest tenth would be 45.7. It would be wrong to write 45.70, because the zero suggests that the number has been rounded to the nearest hundredth. Rounded to the nearest hundredth, 45.679 becomes 45.68.

An exact number such as 1 can be written as 1., 1.0, 1.00, etc. However, a rounded number cannot have these zeros added to the right. For example, when 2.95 is rounded to a whole number, we get 3. This rounded 3 is not the same as 3.00.

EXAMPLE

Round 32.698 to the nearest hundredth.

Solution 32.698 becomes 32.70. Here we must keep the last zero, since we were asked to round off to the hundredths place.

RULE

ROUNDING

If the digit to the right of the rounded digit is less than 5, leave the digit the same.
If the digit to the right of the rounded digit is 5 or more, increase the rounded digit by one.
Discard all digits to the right of the rounded digit.

Note, you must always start with the given number; do not round an already rounded number. If you did, your errors would be larger than when you round the original number.
CHAPTER 3  DECIMALS AND PERCENTS

EXAMPLE
Round 3.498 to the nearest (a) tenth, (b) hundredth, (c) whole number.

Solution
(a) 3.498 rounded to the nearest tenth is 3.5.
(b) 3.498 rounded to the nearest hundredth is 3.50.
(c) 3.498 rounded to the nearest whole number is 3.

EXAMPLE
Round 0.574 to the nearest (a) tenth, (b) hundredth, (c) whole number.

Solution
(a) 0.574 becomes 0.6 (7 > 5).
(b) 0.574 becomes 0.57(4 < 5).
(c) 0.574 becomes 1.

In practical applications we are not told when to round a number. Numbers representing money are usually rounded to the nearest cent.

A general rule for rounding is that we round the answer so that it has the same number of digits as the numbers given in the problem. For example, a whole number divided by a whole number is a whole number. An answer in decimal form is rounded to the fewest decimals in the numbers you have worked with.

EXERCISE 3.1.3
Round to the nearest tenth, hundredth, and whole number.

1. 45.943
2. 0.845
3. 3.899
4. 0.08
5. 0.967
6. 1.529
7. 5.282
8. 10.625
9. 8.624
10. 15.975

3.2  BASIC OPERATIONS WITH DECIMALS

Addition and Subtraction

The procedure for adding and subtracting numbers containing decimal points is the same as for whole numbers. We line up the numbers with the same place values in the same column. The easiest way to do this is to line up the decimal points. (Remember, if there is no decimal point, it is understood to be at the end of the number. 2 = 2., for example.)
PART I ARITHMETIC

EXAMPLE
Add: 2 + 0.3 + 1.15 + 0.009

Solution
\[
\begin{array}{c}
2. \\
0.3 \\
1.15 \\
+ 0.009 \\
\end{array}
\]

You are allowed to fill in the empty spaces with zeros if you wish. Many people think it is easier to rewrite the problem like this:

\[
\begin{array}{c}
2.000 \\
0.300 \\
1.150 \\
+ 0.009 \\
\end{array}
\]

\[3.459\]

In examples in math books, the numbers are supposed to be exact (not rounded). In real life the numbers are rounded before addition.

In subtraction it is advisable to fill in the zeros in cases like 3 – 0.04, which then becomes

\[
\begin{array}{c}
3.00 \\
- 0.04 \\
\end{array}
\]

\[2.96\]

Otherwise it is easy to get lost when you borrow. You can ignore the zeros at the end of the decimals, but you cannot ignore the zeros that are between the decimal and the first nonzero digit. 0.04 is equal to .04 and also to 0.0400, but not to 0.4! Practice addition and subtraction on paper, and then check the answers by calculator.

RULE

ADDITION AND SUBTRACTION OF DECIMAL NUMBERS
1. Place the numbers with the decimal points below each other.
2. Proceed as with whole numbers.

EXAMPLE
Add: (a) −2.3 + 4.5  (b) −0.43 + (−0.2)

Solution  Follow the rules for addition of integers.
(a) Subtract absolute values: 4.5 − 2.3 = 2.2. The sign of the answer is the sign of the larger absolute value.
−2.3 + 4.5 = 2.2

(b) Add absolute values:
\[
\begin{array}{c}
0.43 \\
+ 0.20 \\
\end{array}
\]

\[0.63\]
The sign of the sum is the sign of the numbers added.

\[-0.43 + (-0.2) = -0.63\]

**EXAMPLE**

Subtract: (a) \(13 - 0.4\) \hspace{1cm} (b) \(0.59 - 1\)

**Solution**

(a) Place the numbers so that the decimal points line up.

\[
\begin{array}{c}
13.0 \\
-0.4 \\
12.6
\end{array}
\]

(b) Subtract absolute values.

\[
\begin{array}{c}
1.00 \\
-0.59 \\
0.41
\end{array}
\]

The sign of the difference is the sign of the largest absolute value. \(0.59 - 1 = -0.41\).

**EXERCISE 3.2.1**

Solve.

1. \(1 + 2.4\)
2. \(16 + 3.8\)
3. \(12.34 + 8.85 + 9.3 + 10.2\)
4. \(0.099 + 0.93 + 1.2\)
5. \(15 + 2.9 + 0.87 + 4.583\)
6. \(3.4 - 2.9\)
7. \(12 - 8.63\)
8. \(15.09 - 12\)
9. \(0.99 - 0.099\)
10. \(16 - 8.5 + 2.36 - 3.75\)
11. \(1.15 - (-2.3)\)
12. \(-13.18 - 45.32\)
13. \(4.00 - 0.004 - 4.02\)
14. \(-0.179 - 2.85 - (-0.0003)\)
15. \(2.007 - 32.41 - 91.63 - (-0.016)\)
16. \(81.369 - 7.45 + (-13.026) - (-11.2)\)
PART I ARITHMETIC

Multiplication and Division by Powers of 10

Look at these examples:

\[ 5 \times 10 = 50 \]
\[ 5 \times 100 = 500 \]
\[ 5 \times 1000 = 5000 \]

What will the answer be when we multiply 5 by 100,000?

We know that division is the opposite of multiplication. Now look at these examples:

\[ 50 \div 10 = 5 \]
\[ 500 \div 100 = 5 \]
\[ 5000 \div 1000 = 5 \]

Can you make a general rule for multiplication and division by powers of 10 (10, 100, 1000, etc.)?

We have mentioned that when we multiply a whole number by 10, we add a 0 to the right of the number. This is the same as moving the number one place to the left (2 \times 10 = 20). We can just as easily move the decimal point one step to the right.

\[ 2.00 \times 10 = 20. \]
\[ 0.2 \times 10 = 2. \]
\[ 0.03 \times 10 = 0.3 \]

To multiply by 100, we add two zeros to the right of the whole number or move the decimal point two steps to the right.

\[ 2.00 \times 100 = 200. \]
\[ 0.2 \times 100 = 20. \]
\[ 0.002 \times 100 = 0.2 \]

To divide by 10, we move the decimal point to the left.

\[ 200. \div 10 = 20. \]
\[ 20. \div 10 = 2. \]
\[ 2. \div 10 = 0.2 \]

To divide by 100, we move the decimal point two steps to the left:

\[ 200. \div 100 = 2. \]
\[ 2. \div 100 = 0.02 \]

EXAMPLE

(a) 2500 \div 1000 \hspace{1cm} (b) 2500 \div 10,000

Solution \hspace{0.5cm} (a) 2500 \div 1000 = 2.5 \hspace{1cm} (b) 2500 \div 10,000 = 0.25

Since \[ 10 = 10^1, \hspace{0.5cm} 100 = 10^2, \hspace{0.5cm} 1000 = 10^3, \] and so on, we can state a general rule for multiplying and dividing by powers of 10.

RULE

MULTIPLICATION AND DIVISION BY POWERS OF 10

To multiply a number by 10 raised to a whole number, move the decimal point to the right the same number of steps as the exponent.

To divide a number by 10 raised to a whole number, move the decimal point to the left the same number of steps as the exponent.
CHAPTER 3  DECIMALS AND PERCENTS

EXERCISE 3.2.2

Solve by multiplying or dividing in your head. Check your answer with the calculator.
1. 0.2 × 100
2. 0.035 × 10
3. 7583 ÷ 1000
4. 572 ÷ 100
5. 0.00239 × 10,000
6. 0.006 × 1,000,000
7. 17.8 ÷ 100
8. 950 ÷ 100,000

Multiplication

We know already that multiplication and division are related.

\[
\begin{align*}
10 \times 0.1 &= 1 \\
100 \times 0.01 &= 1 \\
1000 \times 0.001 &= 1 \\
10 \div 10 &= 1 \\
100 \div 100 &= 1 \\
1000 \div 1000 &= 1
\end{align*}
\]

EXAMPLE

Multiply: (a) 300 × 0.01  (b) 4562 × 0.0001

Solution
(a) Instead of multiplying by 0.01, we can divide by 100. Move the decimal point two places to the left.

\[
300 \times 0.01 = 3.00 = 3
\]

(b) Instead of multiplying by 0.0001, we can divide by 10,000. Move the decimal point four steps to the left.

\[
4562 \times 0.0001 = 0.4562
\]

What if we want to multiply 4 × 0.03? We could use addition:

\[
0.03 + 0.03 + 0.03 + 0.03 = 0.12
\]

But it would be more convenient to change 0.03 to 3 × 0.01 and get

\[
4 \times 0.03 = 4 \times 3 \times 0.01 = 12 \times 0.01 = 0.12
\]  (Move the decimal point two places to the left.)

EXAMPLE

Multiply: (a) 13 × 0.2  (b) 156 × 0.004

Solution
(a) 13 × 0.2 = 13 × 2 × 0.1 = 26 × 0.1 = 2.6
(b) 156 × 0.004 = 156 × 4 × 0.001 = 624 × 0.001 = 0.624
PART I ARITHMETIC

When you multiply two decimal numbers, such as $1.25 \times 0.03$, you can first ignore the decimals and multiply $125 \times 3$, which is 375. Then count the total number of decimal places in the original problem and move the decimal point that many places to the left. In this case you move the decimal point $2 + 2 = 4$ places to the left:

$$0.0375 \text{ or } 0.000375$$

Let's analyze what we are actually doing:

1.25 is the same as $125 \times 0.01$

and

0.03 is $3 \times 0.01$

Therefore,

$$1.25 \times 0.03 = 125 \times 0.01 \times 3 \times 0.01$$

$$= 125 \times 3 \times 0.01 \times 0.01$$

$$= 375 \times 0.01 \times 0.01$$

$$= 375 \times 0.0001$$

$$= 0.0375$$

(Multiplication is commutative, so we can switch the order of the factors.) Multiplication by 0.0001 tells us to move the decimal point four places to the left.

EXAMPLE

Multiply: (a) $3.2 \times 0.04$  (b) $15.04 \times 0.003$

Solution

(a) $3.2 \times 0.04 = 32 \times 0.1 \times 4 \times 0.01 = 128 \times 0.1 \times 0.01$

Move the decimal point a total of three places to the left:

$$3.2 \times 0.04 = 0.128$$

(b) $15.04 \times 0.003 = 1504 \times 0.01 \times 3 \times 0.001 = 4512 \times 0.01 \times 0.001$

Move the decimal point $2 + 3 = 5$ places to the left:

$$15.04 \times 0.003 = 0.04512$$

You should, of course, continue to use the shortcut of counting the decimals (or use a calculator) when you multiply, but it is often good to understand why and not only how we determine the position of the decimal point.

RULE

MULTIPLICATION OF DECIMAL NUMBERS

1. Ignore decimals and multiply as with whole numbers.

2. Place a decimal point in the product. The number of decimal places in the product is the sum of the number of decimal places in the factors.
CHAPTER 3  DECIMALS AND PERCENTS

EXAMPLE

Multiply: (a) $1.3 \times 3.4$  \hspace{1cm} (b) $2.45 \times 0.06$

Solution

(a) $13 \times 34 = 442$

There are two decimal places, so $1.3 \times 3.4 = 4.42$.

(b) $245 \times 6 = 1470$

In the original problem, there were four decimal places, so

$2.45 \times 0.06 = 0.1470 = 0.147$ (rounded)

When you multiply using a calculator, it is important to estimate the answer in your head. It is very easy to forget to press down the decimal point key; your answer could be 10 or 100 times too big or too small—or even more!

In order to estimate an answer, we first round off the number we are using. For example, $0.4 \times 3.9$ is approximately $0.4 \times 4 = 1.6$. A calculator gives the correct answer of $1.56$.

$0.8 \times 7.3$ can be estimated as $1 \times 7$. The correct answer is $5.84$, but our value is good enough for a quick approximation. If you had missed one or both of the decimals, your answer would have been $58.4$ or $584$. The estimate of $7$ would help you recognize an error in solving the problem.

EXAMPLE

Estimate: $29 \times 0.13$

Solution  \hspace{1cm} 29 \times 0.13 should be estimated as $30 \times 0.1 = 3$, not $30 \times 0 = 0$. Do not use zero as a factor when you estimate a product. Imagine that you need to estimate how many box cars of wheat you need to ship. The answer 3 is very different from 0!

In the following problems, first estimate the answer, then do it by hand and also by calculator. If you still have trouble with any multiplication facts, review Chapter 1. Estimating the answer is extremely important as a check. You don’t want to lose hundreds of dollars because the decimal point is in the wrong place.

Become comfortable with developing the “hunches” that can be warning messages when you make mistakes in calculations. You don’t need to do long calculations by hand, but you do need to analyze the problems to get an idea of what the answer should look like.

EXERCISE 3.2.3

Estimate the answer, do the work by hand, then use your calculator.

1. $5.3 \times 4.8$
2. $0.032 \times 0.25$
3. $1.05 \times 0.39$
4. $0.0009 \times 5,000,000$
5. $42.9 \times 51.4$
6. $4.29 \times 0.15$
7. $16.3 \times 0.87$
8. $278.9 \times 45.5$
9. $39.96 \times .0062$
10. $0.0015 \times 0.0053$
PART I ARITHMETIC

Division
We described long division with whole numbers in Chapter 1.

EXAMPLE
Solve: $1.16 ÷ 2$

Solution  Fill in the “box” first with 1.16. Now we have 2 into 1.16, or $2|1.16$. Mark the decimal point above the division box, and then divide:

$$
\begin{array}{c}
2 \overline{)1.16} \\
\underline{-1.0} \\
0.16 \\
\underline{-16} \\
0
\end{array}
$$

$4000 ÷ 400$ is the same as $400 + 40$ or $40 + 4$. It is also the same as $4 + 0.4$ and $0.4 + 0.04$. (As you can see when you check this with your calculator, the answer in all cases is 10.)

Similarly, $4.56 ÷ 0.4$ is the same as $45.6 + 4$. Since it is easier to divide by a whole number than with a decimal, we always change the outer number (divisor) to a whole number, and at the same time we move the decimal point of the inner number (the dividend) the same number of steps.

EXAMPLE
Divide: $4.56 ÷ 0.4$

Solution  We have $0.4 \overline{)4.56}$

To change 0.4 to a whole number, we move the decimal point one step to the right. When we move the decimal point in the divisor, we must move it the same number of places in the dividend. Thus,

$$
0.4 \overline{)4.56} = 4 \overline{)45.6}
$$

Now long division! First mark the decimal point for the answer so it doesn’t get lost:

$$
\begin{array}{c}
4 \overline{)45.6} \\
\underline{-4} \\
11 \overline{)45.6} \\
\underline{-4} \\
5 \overline{)45.6} \\
\underline{-4} \\
16 \overline{)45.6} \\
\underline{-16} \\
0
\end{array}
$$

EXAMPLE
Solve: (a) $3.5 ÷ 0.02$  (b) $16.48 ÷ 1.6$
CHAPTER 3  DECIMALS AND PERCENTS

Solution

(a) \[ 3.5 + 0.02 \rightarrow \frac{175.}{0.2} = 3.50 \]

\[ \begin{array}{c}
-2 \\
15 \\
-14 \\
10 \\
-10 \\
0 \\
\end{array} \]

(b) \[ 16.48 + 1.6 \rightarrow 1.6|16.48 \rightarrow 16)|164.8 \]

\[ \begin{array}{c}
-16 \\
48 \\
-48 \\
0 \\
\end{array} \]

RULE

DIVISION OF DECIMAL NUMBERS

1. Move the decimal point in the divisor to obtain a whole number.
2. Move the decimal point the same number of places in the dividend.
3. Proceed as with whole numbers.
4. Mark the decimal point in the quotient directly above the decimal point
   in the dividend.

Most of the time you will not want to do long division by hand but will prefer
to use your calculator instead. As with all calculator work, it is important that
you estimate your answer. Look at the numbers before you start. In the example
above, 3.5 ÷ 0.02, we want to know how many times 0.02 goes into 3.5. 0.02 is a
small number, so the answer must be pretty large. It is approximately 200.

In 35.1 ÷ 0.3, for example, we suggest that you first estimate the answer by
thinking: 35.1 is close to 30, and 30 divided by 0.3 is the same as 300 divided by
3, or 100.

Observe that here we round off 35.1 to 30 for convenience (to divide by 0.3)
instead of the usual 40. You could instead round 35.1 to 36 and get 360 + 3, which
gives 120 as an estimate; both 100 and 120 are fine as estimates.

By hand you would get

\[ \begin{array}{c}
0.3)35.1 \rightarrow 3)351 \rightarrow 3)351 \\
-3 \\
-5 \\
-3 \\
-21 \\
-21 \\
\end{array} \]

\[ \begin{array}{c}
17 \\
0 \\
\end{array} \]
PART I ARITHMETIC

EXAMPLE

Divide: 43.45 ÷ 0.18

Solution  43.45 ÷ 0.18 is approximately 40 ÷ 0.2 or 400 ÷ 2, which is 200. The correct answer is 241.39.

EXERCISE 3.2.4

For practice, estimate the answer. Then do the division on paper and check with your calculator. As a matter of routine, do the problem twice when you use a calculator. It is very easy to make mistakes.

1. 2 ÷ 0.5
2. 5 ÷ 0.2
3. 4.54 ÷ 0.04
4. 1.32 ÷ 0.03
5. 0.025 ÷ 2
6. 0.063 ÷ 3
7. 2.1 ÷ 0.02
8. 55 ÷ 0.005
9. 2.8 ÷ 0.014
10. 56 ÷ 0.005
11. 11.52 ÷ 0.9
12. 23.67 ÷ 0.09
13. 0.045 ÷ 0.0009
14. 135.95 ÷ 2.719

Terminating and Nonterminating Decimals

In the examples so far, the division came out exactly. This is often not the case. In elementary school, most of us learned to write the “remainder” when the division did not come out exactly. For example, 46 ÷ 7 would give an answer of 6 R4. In practical life we never use this notation. When the quotient in a division comes out exactly, such as in 1 ÷ 5 = 0.2, we have a terminating decimal number. However, in 1 ÷ 6, for example, we could continue the division forever. In mathematics we say that we have a nonterminating decimal number. The notation for a decimal that never ends is either . . . , which we already know means “goes on forever,” or a bar above the repeating decimal. Here the digit 6 is repeated, so this decimal number is also repeating.

0.1666 . . . can be written 0.16

The quotient of any two integers is either an integer, a terminating decimal, or a repeating nonterminating decimal number. There exist numbers that are nonterminating and also nonrepeating but they cannot be found by division of two integers. They will be discussed briefly in the next section.

EXAMPLE

Divide: (a) 1 ÷ 6  (b) 46 ÷ 7  (c) 1 ÷ 8
CHAPTER 3 DECIMALS AND PERCENTS

Solution

(a) \(1 + \frac{6}{6}\) 
\[
\begin{array}{c}
1.0000 \\
- 6 \\
- 40 \\
- 36 \\
- 40 \\
- 36 \\
- 4 \\
\end{array}
\]
= \(0.1\overline{6}\) Nonterminating (repeating)

(b) \(46 + \frac{7}{7}\) 
\[
\begin{array}{c}
46.000000000 \\
- 42 \\
- 40 \\
- 35 \\
- 50 \\
- 49 \\
- 10 \\
\end{array}
\]
= \(6.57142857\overline{1}\) Nonterminating (repeating)

(c) \(1 + \frac{8}{8}\) 
\[
\begin{array}{c}
1.0 \\
- 8 \\
- 20 \\
- 16 \\
- 40 \\
- 40 \\
\end{array}
\]
= \(0.125\) Terminating

EXERCISE 3.2.5

Divide by hand and check with calculator.

1. \(0.1 + 1.8\)
2. \(2.4 + 0.72\)
3. \(0.05 + 0.6\)
4. \(4.9 + 0.77\)
5. \(2 + 1.1\)
6. \(0.1 + 0.07\)
7. \(3.56 + 4.8\)
8. \(0.99 + 6.5\)

Irrational Numbers

Some numbers are both nonterminating and nonrepeating. Examples of such numbers are \(\pi\) (pi) and \(\sqrt{2}\). They are called irrational numbers.

The value of \(\pi\) cannot be expressed as an exact decimal number. To seven decimal places, \(\pi \approx 3.1415926\), but even that is not exact. However, for our work we can use the approximation \(\pi \approx 3.14\). Mathematicians have calculated \(\pi\) to many decimal places and are still working to determine more.
PART I ARITHMETIC

√2 also cannot be expressed as either a terminating decimal or a nonterminating decimal with repeating groups. √2 is approximately equal to 1.4. A better approximation is 1.4142, the square of which equals 1.99996.

EXAMPLE

Find a decimal approximation for √15.

Solution We know that √16 = 4, so √15 < 4

<table>
<thead>
<tr>
<th>Guess</th>
<th>√15 ≈</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.9</td>
<td>3.9² = 15.21</td>
<td>Too high</td>
</tr>
<tr>
<td></td>
<td>3.85</td>
<td>3.85² = 14.83</td>
<td>Too low</td>
</tr>
<tr>
<td></td>
<td>3.88</td>
<td>3.88² = 15.05</td>
<td>O.K.</td>
</tr>
</tbody>
</table>

By calculator we get √15 ≈ 3.8729833, which we can round to 3.87.

EXERCISE 3.2.6

Approximate to two decimal places by trial and error.

1. √8
2. √13
3. √18
4. √26

Applications

Here are some word problems from an 1858 arithmetic book (edited slightly for today's language and spelling).

EXERCISE 3.2.7

1. Bought 14.75 yards of sheeting at 14 cents per yard. What was the cost of the piece? Express answer in dollars.
2. Bought land at $62.50 per acre, and then sold it at $75 per acre, thereby making $846.875. How many acres were bought?
3. How much will 5625 feet of lumber cost at $15.525 per thousand feet? (Round answer to nearest hundredths.)
4. How many bushels of onions at $0.82 per bushel can be bought for $112.24? (Round answer to a whole number.)
5. A merchant deposits $687.25 in a bank, and then later he deposits $943.64. If he draws out $875.29, how much will remain in the bank?

The following problems are more recent.

6. On successive holes a golfer drives a golf ball 205.4, 197.5, 182.75, and 220.25 yards. Find the total number of yards on the four holes.
7. A tank holds 300 gallons. If a pipe empties 0.25 of the tank in 1 hour, how many gallons will be left at the end of 2 hours?
8. If 0.1 inch on a map represents 49 miles, how many miles are represented by 3 inches on the map?
CHAPTER 3 DECIMALS AND PERCENTS

9. If a steel tape expands 0.00016 in. for each inch when heated, how much will a tape 100 feet long expand?

10. If your car averages 37 miles per gallon, approximately how many gallons will you need to drive across the United States from San Francisco to New York City (2934 miles)? Is $400 enough to budget for gas across and back assuming a gas price average of $1.95 a gallon?

3.3 PERCENTS

Conversions from Percent to Decimal

The word “percent” means “per hundred.” Ten percent (10%) is 10 per 100, for instance. One hundred percent (100%) is 100 per 100, so it is the “whole thing” or 1. “Per” tells us to divide. To change a percent into a decimal, replace “%” with “divided by 100.” This in turn leads us to move the decimal point two steps to the left.

EXAMPLE

Express as a decimal:

(a) 1%  (b) 10%  (c) 100%  (d) 1000%  (e) 5%  (f) 35%
(g) 0.4%  (h) 216%

Solution

(a) 1% = 0.01  (b) 10% = 0.10  (c) 100% = 1  (d) 1000% = 10
(e) 5% = 0.05  (f) 35% = 0.35  (g) 0.4% = 0.004  (h) 216% = 2.16

RULE

To convert a percent into a decimal, divide by 100. Move the decimal point two steps to the left.

EXERCISE 3.3.1

Convert percent to decimal.

1. (a) 25%  (b) 75%
2. (a) 2.5%  (b) 8.25%
3. (a) 200%  (b) 743%
4. (a) 0.3%  (b) 0.5%
5. (a) 0.01%  (b) 0.05%

Conversions from Decimal to Percent

Any number can be written as a percent. 0.5 is equal to 50%. (Working backwards we see that 50% = 50/100 = 0.5. Similarly, 0.01 = 1%. Again we see that 1% = 1 + 100 = 0.01.)
PART I ARITHMETIC

To change any number into a percent, simply multiply by 100%. For example, \(5 = 500\%\). To multiply a decimal number by 100, we move the decimal point two steps to the right:

\[
\begin{align*}
0.02 &= 0.02 \times 100\% = 2\% \\
0.2 &= 0.2 \times 100\% = 20\% \\
2 &= 2 \times 100\% = 200\%
\end{align*}
\]

RULE

To express any number as a percent, multiply it by 100%. Move the decimal point two steps to the right.

EXERCISE 3.3.2

Express the number as a percent.

1. (a) 0.25    (b) 0.68
2. (a) 1.3      (b) 1.05
3. (a) 5       (b) 200
4. (a) 2.5      (b) 6.2
5. (a) 0.0075   (b) 0.0091

"Percent of"

Most of the time when we use percent, we deal with the "percent of" something. The tip is 15% of the total bill. The security deposit is 150% of the monthly rent. 40% of the voters turned out for the election. The couple donated 10% of their income to charity.

Many percent problems can be rewritten in the form

A certain percent of a number equals what number?

That is, we know the percent and one number, and we need to find the other number. In solving such problems, we recognize that the phrase "percent of" tells us to multiply by the percent.

EXAMPLE

50% of 200 is what number?

Solution  First guess the answer. You may already know that 50% is half, so 50% of 200 is 100. But what if you don't? This problem tells you to multiply 50% by 200. The easiest way to do this is to convert 50% to a decimal number, then multiply. Thus,

\[
50\% \text{ of } 200 = 0.50 \times 200 = 100
\]

EXAMPLE

What is 15% of 35?
Solution  Try to guess the answer. How much tip would you leave on a $35 restaurant check? About $5.00? We change 15% to the decimal number 0.15. Now,

\[ 0.15 \times 35 = 5.25 \]

So if you liked the service, you’d probably leave a little more than $5.

Sometimes we are asked to solve for the percent. What percent of the voters turned out for the election? If you were charged $3.56 tax on an $89 tape deck, what percent tax were you charged? If you received a $120 raise on a monthly salary of $1600, what percent raise did you receive?

Here the percent problem can be written in the form

\[
\text{What percent of a number equals another number?}
\]

In this case, we have the two numbers and solve to find the percent. For example, what percent of 200 is 100? We know from the first example above that 50% of 200 = 100. Figure 3.2 helps illustrate this problem.

\[
\begin{array}{c|c|c}
200 & 100 & 100 \\
\hline
50\% & + & 50\% \\
\hline
100\% & & \\
\end{array}
\]

Figure 3.2

We see that when 200 is split into two equal parts, each part is 100, and when 100% is split into two equal parts, each part is 50%.

We can find the percent by treating this problem as a straightforward division problem using decimal numbers. In the case of “what percent of 200 is 100?,” we put the problem in the form “100 is what percent of 200?” and divide 100 by 200.

\[
100 + 200 = 0.50
\]

Now we convert 0.50 to a percent by multiplying it by 100%:

\[
0.50 \times 100\% = 50\%
\]

EXAMPLE

What percent of 20 is 5?

Solution  This fits the form: 5 is what percent of 20? This leads us to

\[
5 + 20 = 0.25 = 25\%
\]

The figure shows that if the whole (100%) is 20, then 5 represents 25%.

\[
\begin{array}{c|c|c|c|c|c}
20 & 5 & 5 & 5 & 5 & 100\% \\
\hline
25\% & 25\% & 25\% & 25\%
\end{array}
\]
PART I ARITHMETIC

EXAMPLE
What percent of 40 is 8?

Solution  \[ 8 + 40 = 0.20 = 20\% \]
Again, the figure shows that if 100\% is 40, then 8 is 20\%.

<table>
<thead>
<tr>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
</tr>
<tr>
<td>20%</td>
</tr>
</tbody>
</table>

EXAMPLE
What percent of 80 is 20?

Solution  \[ 20 + 80 = 0.25 = 25\% \]

EXAMPLE
What percent of 230.60 is 40.355?

Solution  \[ 40.355 \div 230.60 = 0.175 = 17.5\% \]

EXAMPLE
If you received a $120 raise on a monthly salary of $1600, what percent raise did you receive?

Solution  What percent of $1600 is $120?
\[ 120 \div 1600 = 0.075 = 7.5\% \]

The raise in salary was 7.5\%.

A third type of percent problem takes the form: A certain percent of what number equals another number.

In this case, we know the percent and the resulting number, and need to find the number we are taking the percent of. For example, if you know the interest rate and the interest, how much money do you have invested?

EXAMPLE
How much money should be deposited to earn interest of $1000 in 1 year if the interest rate is 2%?

Solution  This is really asking "$1000 is 2\% of what amount?" Draw a picture.

<table>
<thead>
<tr>
<th>$1000</th>
<th>$1000</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>2%</td>
<td>2%</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2%</td>
</tr>
</tbody>
</table>

\[ 100\% \]

100\% + 2\% = 50 and 50 \times $1000 = $50,000

You need to deposit $50,000 to earn $1000 interest per year.
EXAMPLE
If the mortgage company requires a 20% down payment and you have $16,000 available, what is the maximum price you can afford for a home?

**Solution**  Even though you’ll pay the $16,000 only once, the following diagram can help solve the problem:

<table>
<thead>
<tr>
<th></th>
<th>$16,000</th>
<th>$16,000</th>
<th>...</th>
<th>$16,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td></td>
<td>20%</td>
<td>...</td>
<td>20%</td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$16,000 + 20\% = 5 \quad \text{and} \quad 5 \times 16,000 = 80,000$

We will need a quick way of solving this type of percent problem. We can do this using division. This time, however, we will divide by the percent, after it has been converted to a decimal number. Thus, in an interest example, “5% of what number” leads us to

\[
1000 + 5\% = 1000 + 0.05 = 20,000
\]

**EXAMPLE**
50% of what number is 2?

**Solution**  \[2 \div 50\% = 2 \div 0.50 = 4\]

**EXAMPLE**
25% of what number is 3?

**Solution**  \[3 \div 25\% = 3 \div 0.25 = 12\]

**EXAMPLE**
10% of what number is 6?

**Solution**  \[6 \div 10\% = 6 \div 0.10 = 60\]

**EXAMPLE**
18.5% of what number is 61.05?

**Solution**  \[61.05 \div 18.5\% = 61.05 \div 0.185 = 330.00\]

The following example shows the relationship of these three forms.

**EXAMPLE**
(a) 39% of 46 is what number?
(b) What percent of 46 is 17.94?
(c) 39% of what number is 17.94?

**Solution**
(a) \[39\% \times 46 = 17.94\]
(b) \[17.94 \div 46 = 0.39 = 39\%\]
(c) \[17.94 \div 39\% = 17.94 \div 0.39 = 46\]
PART I ARITHMETIC

RULE

SOLVING A PERCENT PROBLEM
Rewrite the problem in one of these forms:
1. A certain percent of a certain number equals what number?
   Solve by multiplication.
2. What percent of a certain number equals another number?
   Solve by division.
3. A certain percent of what number equals another number?
   Solve by division.

EXERCISE 3.3.3
Translate each problem into one of the forms of "some percent of a number is another number" and solve. Round your answers to the nearest hundredth.

1. 8 is what percent of 32?
2. $13 is what percent of $6.50?
3. 1% of what number is 30?
4. 25% of what number is 8?
5. Find 12.5% of 40.
6. Find 7.4% of 195.
7. 12 is what percent of 10?
8. 24% of what number is 120?
9. 50 is 125% of what number?
10. 0.4% of what number is 5?
11. The enrollment at a local community college is 3500. Of these, 30% are liberal arts majors. How many liberal arts majors are there at the college?
12. Lisa spends $175 of her monthly take-home pay for rent. If her monthly salary is $700, what percent does she spend for rent?
13. Suppose 84 people out of 150 interviewed planned to vote for the Democrats in the next election. What percent is that?
14. Suppose 120 students out of 150 passed a chemistry course. What percent is that?
15. Earl lost 10 pounds in 3 months from his original weight of 160 pounds. Find the percent of his decrease in weight.
16. The sales tax on a used car is $294. If the purchase price is $4200, find the sales tax rate.
17. If the sales tax is $1.17 on a purchase of a book priced at $29.25, find the sales tax rate.
18. The price of a cookbook is reduced 12%. If the discount is $1.44, find the original price of the book.
19. Ms. Smith announced that the average test score on a 25-question test was 72%. How many correct answers is this on the average?
20. 5 out of 2 million people with lottery tickets won. What percent is that?
SPECIAL NOTE ON CALCULATORS

Working with percent on calculators can be tricky. Some calculators have special percent keys; others do not. In either case, you can always convert a percent to a decimal number and continue as you would with any other decimal number. (Be sure to convert final answers back to percent, if needed.)

Even the calculators with percent keys have different ways of using them, depending on the internal programming for that model of calculator. If you wanted to solve $100 + 10\% - 10\%$, you might get two different answers. We know that $10\% = 0.10$, so

$$100 + 10\% - 10\% = 100 + 0.10 - 0.10 = 100$$

However, some calculators read $100 + 10\%$ as $100 + 10\%$ of 100, which equals

$$100 + 0.10 \times 100 = 100 + 10 = 110$$

Now we try to subtract $10\%$ and the calculator is programmed to calculate

$$110 - (10\% \text{ of } 110) = 110 - 11 = 99$$

Estimating the Answer

With the common use of calculators in jobs and school, there has been a decline of skills in mental arithmetic. As we have pointed out several times, it is important to be able to approximate answers quickly in real life. For example, there are known cases of patients in hospitals who have died because the medicine they were given was ten times too strong!

Complete the following example as fast as you can in your head. Check your answers by reworking them with the calculator.

EXAMPLE

Approximate: (a) $253.4 \times 0.05$  (b) $568.9 + 6.7$.

Solution

(a) $0.05$ is much smaller than $0.1$, so the product must be smaller than $25$, and we also know it must be larger than $2.5$ ($0.01 \times 253.4 = 2.53$). The calculator answer is 12.67.

(b) $568.9 + 6.7$ should be around $100$ ($600 + 6$); actually it must be somewhat smaller than $100$ because $568.9$ is less than $600$ and $6.7$ is more than $6$.

The calculator answer rounds to $84.9$.

EXERCISE 3.3.4

Choose the answer closest to the correct answer.

1. $106.9 + 1.5$  (a) 0.07  (b) 0.7  (c) 7  (d) 70
2. $48 + 0.002$  (a) 240  (b) 2400  (c) 24,000  (d) 240,000
3. $304.2 \times 0.16$  (a) 0.06  (b) 0.6  (c) 6  (d) 60
4. $0.03\% \times 50$  (a) 0.015  (b) 0.15  (c) 1.5  (d) 15
5. $0.35 + 60$  (a) 0.0005  (b) 0.005  (c) 0.05  (d) 0.5
PART I ARITHMETIC

6. 0.88 + 2.2 (a) 0.04 (b) 0.4 (c) 4 (d) 40
7. 0.04 × 500 + 0.25 (a) 0.8 (b) 8 (c) 80 (d) 800
8. 0.2% × 5,000,000 (a) 1000 (b) 10,000 (c) 100,000 (d) 1,000,000
9. 5.1 + 0.003 (a) 1.7 (b) 17 (c) 170 (d) 1700
10. 0.25 + 0.5 + 5 (a) 0.001 (b) 0.01 (c) 0.1 (d) 1

Applications

Sometimes we must use addition or subtraction to find information needed in problems involving percent. This is especially true in problems with sales discounts or mark-ups.

EXAMPLE

A dress tag was changed from $80 to $60. What was the discount rate?

Solution Rate is the percent, so we are asked to solve the following problem: What percent of 80 is the discount?
The discount in dollars is the difference between the original price and the sale price. In this case, the discount is $80 – $60 = $20. The problem now becomes

What percent of 80 is 20?

So we have

20 + 80 = 0.25 = 25%

The dress is discounted 25%.

EXAMPLE

A cassette tape sold for $7.50. The price was then reduced by 20%. What was the new price of the tape?

Solution Here we have two possible solution strategies.
First, 20% of 7.50 is 1.50. Since the reduction is 1.50, the sale price of the tape is $7.50 – $1.50 = $6.00.
Second, since there is a 20% reduction, we have to pay 80%.
80% of $7.50 = 0.80 × $7.50 = $6.00

Choose the method you find more comfortable.

EXERCISE 3.3.5

1. A house originally sold for $290,000 but was reduced to $284,000. Find the percent discount.
2. Rebecca earns $40,000 per year as an accountant. If 28% of her salary is withheld for taxes, what is her take-home pay?
3. Fran is a nurse with an annual salary of $35,000. If she receives a 6% raise, what is her new salary?
4. Malcolm’s monthly salary as a teacher increased from $2762 to $2900. Find the percent increase.
5. During a period of one month, the price of eggs rose from $1.15 a dozen to $1.29 a dozen. Find the percent increase to the nearest whole percent.

6. During a period of one month, the price of eggs decreased from $1.29 a dozen to $1.15 a dozen. Find the percent decrease to the nearest whole percent.

7. The enrollment at Coles Junior College increased from 950 to 1080 students. Find the percent increase to the nearest whole percent.

8. During one year the price of a stock went from $87.5 per share to $37.5 per share. Find the percent decrease to the nearest whole percent.

9. Zack paid $7800 for a used car. After 18 months he finds that its value is only $5200. Find the percent decrease.

10. If 36 students out of a school population of 900 were absent one day, what percent were present?

11. A refrigerator was on sale for $430. This gave the store a profit of $70. What was the percent profit to the store?

12. 72% of those taking the driver's test pass. If 2100 took the test, how many failed?

13. Of 2840 entering freshmen, 90% admitted to having math anxiety. How many students did not dread their required math courses?

**SUMMARY**

**Definitions**

Place values: Ones, tenths, hundredths, thousandths, ten-thousandths, . . .

**Rules**

Reading a decimal number: Read the decimal point as “and.” Read the decimal part of the number as a whole number followed by the name of the rightmost place.

Ordering decimal numbers: Arrange the numbers in a column with the decimal points below each other. Compare the place values going from left to right.

Rounding a decimal number: If the digit to the right of the rounded digit is less than 5, leave the digit the same. If the digit to the right of the rounded digit is 5 or more, increase the rounded digit by one. Discard all digits to the right of the rounded digit.

Adding or subtracting decimal numbers: Place the numbers with the decimal points below each other. Proceed as with whole numbers.

Multiplying and dividing by powers of 10: To multiply a number by 10 raised to a whole number, move the decimal point to the right the same number of steps as the exponent.

To divide a number by 10 raised to a whole number, move the decimal point to the left the same number of steps as the exponent.

Multiplying decimal numbers: Multiply as with whole numbers. The number of decimals in the product is the sum of the number of decimals of the factors.

To divide decimals, move the decimal point in the divisor to obtain a whole number. Move the decimal point the same number of places in the dividend. Proceed as with whole numbers. Mark the decimal point in the quotient directly above the decimal point in the dividend.

To change a percent into a decimal, divide by 100. Move the decimal point two steps to the left.